Abstract. Given a de-centralized nature of the development of the Semantic Web, there will be an explosion in the number of ontologies in P2P systems. To integrate data from disparate ontologies, we must know the semantic correspondence between their elements. What we argue is the full epistemic independency of peer databases: they can change their ontology and/or extension of their knowledge independently from other peers and without any communication to other peers. We intend to obtain very robust P2P systems, able to answer to user queries also when intended mappings between peers do not feet with the modified ontologies of peers, but also to have the possibility to map naturally P2P database systems into a grid computation. We introduce S5 modal intensional FOL embedding of P2P systems and consider the real-life query-agents, with a non omniscient deductive model, based on the contextualization of P2P systems. We define the bridge rules for such contextual reasoning, and a query-agent with a ‘belief’ predicate for intensionally equivalent queries over peers, deductively inferred by agent w.r.t. the given user query, and we show that it is sound weakening of the omniscient S5 modal intensional FOL.

1 Introduction

It is well known that the standard epistemic logics used to model the agents suffer from the omniscience problem: that is, an agent is logically omniscient if it knows all the valid formulae, and its knowledge is closed under logical equivalence (or alternatively under logical implication). On the standard epistemic, possible worlds model, we must assume that an agent can derive all consequences of a belief in an instant, i.e., without computational effort.

We investigate why this might be a problem for real-life agents, especially when the standard possible worlds model poses a problem for agents with a big number of large databases, as we will have in Semantic Web applications and P2P databases. Agents may establish immediately certain logical truths or simple consequences of what they consciously assented to. However, there are highly remote dispositional states which could only be established by complex, time-consuming reasoning. The modal framework cannot distinguish between a sentence that an agent consciously assented to and a piece of potential knowledge which could never be made actual by the agent and is therefore not suited to model resource-bounded reasoning.
Whose aim is to develop a usable logic in which they can reason about an agent’s beliefs, commonly distinguish semantic from syntax approaches. The former models an agent beliefs in a modal language with a modal operator for belief, whereas the latter uses a metalanguage, containing a belief predicate, to talk about an agent’s beliefs.

In real Web applications with P2P database systems we will never have omniscient query-agents that will contemporary have the complete and whole knowledge about all ontologies of all peers. Such supposition would generate the system with a global and centralized knowledge, in contrast with a pragmatic and completely decentralized P2P systems, with completely independent peers, which can change their local ontology (database schema) in any instance of time without informing any other peer or “global” system about it.

For such kind of peers we provide a S5 modal intensional First Order Logic, with ‘necessity’ universal modal operator □, for semantic mapping between peers; instead of epistemic modal operator for belief we will use the ‘belief’ predicate of this intensional logic, as in the syntax approach [1,2]. But in our case, instead of using a singular referring term name as in classic extensional FOL, to denote the sentence φ an agent is believing, we will use intensional abstract term which contains the whole structure of the sentence in composite algebraic form (the denotation of such abstract terms is an algebraic formula with the denotations of all subcomponents of the original sentence, that is, it has recursive denotation definition).

We will use also the approach in [3] which takes a set of sentences in the internal language of an agent (in our case it is the language for conjunctive queries), which we can view as initial assumptions (in our case the set of intensional equivalences between views of the actual peer database toward other peers), to comprise agents knowledge base. We combine this approach with the contextual reasoning [4,5,6,7], by considering, for instance, contexts as peer databases: each peer database is a a (local) context for a query-agent: he can access to the local knowledge of this peer, during a query-answering process, and also to the set of local bridge rules [6] of this peer. We can consider the whole P2P database system, composed by a number of peer databases, as a knowledge base which contains a set of interacting contexts $P_1, \ldots P_N$ (peer databases).

The kind of bridge rules, of a context (peer) $P_i$, is of the form:

\[
\frac{v_i : P_i}{v_j : P_j}
\]

where $v_i \approx v_j$ are intensionally equivalent views (conjunctive queries) of a peer database $P_i$ and $P_j$, respectively. This rules allow us to bridge deductions in $P_i$ to deductions in $P_j$ by allowing us to derive $v_j$ in $P_j$ just because we have derived $v_i$ in $P_i$. So for any context, the query agent uses the subset of deduction rules $\rho(P_i)$, derived from a context $P_i$, in order to derive new information: we write $\Gamma \vdash_{\rho(P_i)} \phi$ to mean that $\phi$ is deducible from $\Gamma$ using the rules assigned by $\rho(P_i)$. A formula $\phi$ is said to be believed by an agent, which Konolige writes $B\phi$, where $B$ is a believe modal operator, iff it is in an agent’s belief set $B$, iff it is either in the agent’s initial knowledge base $\Gamma$ or else is derivable form the knowledge base by applying the agent’s deduction rules:

\[(K1) \quad B\phi \iff \phi \in B \quad \text{iff} \quad \phi \in \Gamma \text{ or } \Gamma \vdash_{\rho(P_i)} \phi.\]

In the intensional FOL logic, instead, we will have the predicate form $B(x)$ in the place...
of the Konolige’s belief modal operator $B$, but the derivation process as we will see is analog. As Konolige we assume the deductive closure of an agent’s belief set $B$ w.r.t. the agent’s deduction rules, that is:

$$(K2) \quad \text{If } B \vdash_{\rho(P_i)} \phi \quad \text{and} \quad B \cup \{\phi\} \vdash_{\rho(P_i)} \psi, \quad \text{then} \quad B \vdash_{\rho(P_i)} \psi.$$ 

The property of deductive closure should not be confused with that of logical omniscience: if the deduction rules are logically incomplete, then there will be sentences that are logical consequences of the base set and yet not in the belief set. Thus, deductive closure is much weaker notion than closure under logical consequence of the Intensional FOL theory for P2P system. In what follows we will call it weak deduction: in general, agents modeled in the framework of the Deduction model are not logically omniscient because the rules they use to derive new sentences in their belief set are in some respect incomplete, because they are context-dependent. But if an agent has a complete set of deduction rules, then all logical consequences of an agent’s base beliefs will be in the belief set.

The contextual reasoning has some similarities with the Labelled Deduction System (LDS) [8]. In LDS approach the basic unit of a deductive process are not just formulae but the labelled formulae, where the labels belong to a given “labeling algebra” (which represents the additional information as a separate component of a standard derivation system and can be treated as an independent parameter), and are explicitly incorporated into the object language. In the contextual reasoning we have no explicit labels associated with logic formulae, and even more we have no all inference rules (bridges) which a deductive system can use in its deduction. We have standard logic formulae, but at each context only the subset of context dependent inference rules can be used for a (non labelled) deduction. Also there is no any centralized place where we can see how are partitioned rules in different contexts: thus, the deduction process is similar to the traveling in an unknown country without any global map, by using only local indications (actual context) in order to take the next decisions, which depends on the traveler’s objectives (here it is represented by an user query).

The plan of this paper is the following: After the brief introduction to Intensional FOL language with abstraction operator, in Section 2 we present the S5 modal intensional FOL and we define the intensional equivalence relation, used for the semantic mapping between different databases. In Section 3 we define an embedding of P2P database systems in this intensional FOL: such modal logic has omniscient deductive inference which is not adequate for real-life query-agents in Web based P2P systems. Finally, in Section 4 we define the non omniscient deductive model, based on the contextualization of P2P systems, we define the bridge rules for such contextual reasoning, and a query-agent with a ‘belief’ predicate for intensionally equivalent queries over peers in a P2P system, deductively inferred by agent w.r.t. the given user query over some peer database.

1.1 Introduction to Intensional FOL language with abstraction

Contemporary use of the term ‘intension’ derives from the traditional logical doctrine that an idea has both an extension and an intension. Intensional entities are such things as concepts, propositions and properties. What make them ‘intensional’ is that they violate the principle of extensionality; the principle that extensional equivalence implies
identity. All (or most) of these intensional entities have been classified at one time or another as kinds of Universals [9].

The fundamental entities are intensional abstracts or so called, 'that'-clauses. We assume that they are singular terms; Intensional expressions like 'believe', 'mean', 'assert', 'know', are standard two-place predicates that take 'that'-clauses as arguments. Expressions like 'is necessary', 'is true', and 'is possible' are one-place predicates that take 'that'-clauses as arguments. For example, in the intensional sentence "it is necessary that A", where A is a proposition, the 'that A' is denoted by the \( <A> \), where \( <\cdot> \) is the intensional abstraction operator which transforms a logic formula into a term. So that the sentence "it is necessary that A" is expressed by the logic atom \( N(⋖A⋗) \), where \( N \) is the unary predicate 'is necessary'. In this way we are able to avoid to have the higher-order syntax for our intensional logic language (predicates appear in variable places of other predicates), as, for example HiLog [10] where the same symbol may denote a predicate, a function, or an atomic formula. In the FOL (First-order logic) with intensional abstraction we have more fine distinction between an atom \( A \) and its use as a term "that A", denoted by \( <A> \) and considered as intensional "name", inside some other predicate, and, for example, to have the first-order formula \( \neg A \land P(t, <A>) \) instead of the second-order HiLog formula \( \neg A \land P(t, A) \).

**Definition 1.** The syntax of the First-order Logic language with intensional abstraction \( ⋖\cdot ⋗ \), called \( L_\omega \) in [11], is as follows:

- Logic operators \( (\land, \neg, \exists) \);
- Predicate letters in \( P \) (functional letters are considered as particular case of predicate letters);
- Variables \( x, y, z, \ldots \);
- Abstraction \( ⋖⋗ \), and punctuation symbols (comma, parenthesis).

With the following simultaneous inductive definition of term and formula:

1. All variables are terms.
2. If \( t_1, \ldots, t_k \) are terms, then \( A(t_1, \ldots, t_k) \) is a formula (\( A \in P \) is a k-ary predicate letter).
3. If \( A \) and \( B \) are formulae, then \( (A \land B) \), \( \neg A \), and \( (\exists x)A \) are formulae.
4. If \( A \) is a formula and \( \alpha = \{v_1, \ldots, v_n\} \) is a sequence of distinct variables (a subset of free variables in \( A \)), then \( <A>\alpha \) is a term. The externally quantifiable variables are the free variables not in \( \alpha \). When \( n = 0 \), \( <A> \) is a term which denotes a proposition, for \( n \geq 1 \) it denotes a \( n \)-ary relation-in-intension.

A sentence is a formula having no free variables. The binary predicate letter \( F^2_1 \) is singled out as a distinguished logical predicate and formulae of the form \( F^2_1(t_1, t_2) \) are to be rewritten in the form \( t_1 = t_2 \). The logic operators \( \forall, \lor, \Rightarrow \) are defined in terms of \( (\land, \neg, \exists) \) in the usual way.

For example, "agent believes that A" is given by formula \( B(<A>) \) (\( B \) is unary 'believe' predicate), "Being a bachelor is the same thing as being an unmarried man" is given by identity of terms \( <C(x)>_x = <U(x) \land M(x)>_x \) (with \( C \) for 'bachelor', \( U \) for 'unmarried', and \( M \) for 'man', unary predicates).

## 2 Intensional equivalence and Intensional FOL language

Analogously to Boolean algebras which are extensional models of sentential logic, we introduce an intensional algebra as follows:
**Definition 2.** (SYNTAX): Intensional algebra is a structure 
\[
\text{Alg}_{\text{int}} = \langle D, \text{conj}, \text{disj}, \text{impl}, \text{neg}, \tau, f, t \rangle,
\]
which contains a domain \( D = D_{-1} \cup D_0 \cup D_1 \cup \ldots \cup D_n \), where a subdomain \( D_{-1} \) is made of particulars (extensional entities), and the rest is made of universals (\( D_0 \) for propositions, \( D_n \) for n-ary relations-in-intension (we consider the property as unary relation-in-intension); 

Binary operations in \( \{ \text{conj}, \text{disj}, \text{impl} \} : D_i \times D_i \rightarrow D_i \), for each \( i \geq 0 \), \( \text{pred} : D_i \times D \rightarrow D_{i-1} \), for \( i \geq 1 \), and unary operation \( \text{neg} : D_i \rightarrow D_i \), for each \( i \geq 0 \); 

\( \tau \) is a set of auxiliary operations \([12]\) intended to be semantic counterparts of the syntactical operations of repeating the same variable one or more times within a given formula and of changing around the order of the variables within a given formula; 

\( f, t \) are empty set and \( D \) which may be thought of as falsity and truth, respectively.

**Notice:** in the original work \([12]\) this “algebraization” of the intensional FOL is extended also to logic quantifiers, but for our purpose it is not necessary, so we will use this simpler intensional algebra. Thus, our logic is an intensional predicate logic without quantifiers. It is used for the virtual predicates defined as virtual views over peer databases: the intensional equivalence of these predicates is used by query-agents in order to answer to conjunctive queries, and we will use non omnисient intensional reasoning for such agents.

The distinction between intensions and extensions is important especially because we are now able to have and *equational theory* over intensional entities (as \( \langle A \rangle \)), that is predicate and function “names”, that is separate from the extensional equality of relations and functions. Thus, intensional FOL has the simple Tarski first-order semantics, with a decidable unification problem, but we need also the actual-world mapping which maps any intensional entity to its actual-world extension. In what follows we will identify a possible world by a particular mapping which assigns to intensional entities their extensions in such possible world. That is direct bridge between intensional FOL and possible worlds representation \([13,14,15,16,17]\), where intension of a proposition is a function from possible worlds \( \mathcal{W} \) to truth-values, and properties and functions from \( \mathcal{W} \) to sets of possible (usually not-actual) objects.

The connection between Bealer’s non-reductionistic and Montague’s possible world approach to intensional logic can be given by the isomorphism (each extensionalization function can be taken as a possible world) 

\[
\mathcal{F} : \mathcal{W} \cong \mathcal{E},
\]

where \( \mathcal{E} \) is a set of possible extensionalization functions: Each extensionalization function \( h \in \mathcal{E} \) assigns to the intensional elements of \( D \) an appropriate extension as follows: for each proposition \( x \in D_0 \), \( h(x) \in \{ f, t \} \) is its extension (true or false value); for each n-ary relation-in-intension \( x \in D_n \), \( h(x) \) is a subset of \( D^n \) (n-th Cartesian product of \( D \)); in the case of particulars \( x \in D_{-1} \), \( h(x) = x \). Among the possible functions in \( \mathcal{E} \) there is a distinguished function \( \mathcal{E} \) which is to be thought as the actual extensionalization function: it tells us the actual extension of the intensional elements in \( D \).

**Definition 3.** (SEMANTICS): The operations of the algebra \( \text{Alg}_{\text{int}} \) must satisfy the following conditions, for any \( h \in \mathcal{E}, x_1, \ldots, x_i \in D \):

1.1 \( \langle x_1, \ldots, x_i \rangle \in h(\text{conj}(u, v)) \iff \langle x_1, \ldots, x_i \rangle \in h(u) \cap h(v) \), for \( u, v \in D_i, i \geq 1 \).
Model: A model for the intensional FOL is the S5 Kripke structure

Intensional Equivalence: the two intensional entities

We can connect 𝐴 and 𝛼 of intensional logics with modal Kripke based logics. The correspondence, not present in original intensional theory [9], is a natural identification

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Remark: this semantics is equivalent to the algebraic semantics for 𝐿 𝜔  in [11] for the case of the conception where intensional entities are considered to be identical if and only if they are necessarily equivalent. Moreover, for this intensional FOL holds the soundness an completeness: For all formulae 𝐴 in 𝐿 𝜔  , 𝐴 is valid if and only if 𝐴 is a theorem of this First-order S5 modal logic with intensional equality [11].

Now we can introduce the new intensional equivalence relation between intensional entities:

Definition 5. (Intensional Equivalence): the two intensional entities 𝐴 ≡ 𝐵 are intensionally equivalent if and only if they are necessarily equivalent. Moreover, for this intensional FOL holds the soundness an completeness: For all formulae 𝐴 in 𝐿 𝜔  , 𝐴 is valid if and only if 𝐴 is a theorem of this First-order S5 modal logic with intensional equality [11].

This definition of equivalence relation is the flat-accumulation case presented in [18,19,20]: if the first predicate is true in some world then the second must be true in some world
also, and vice versa. Each equality is also intensional equivalence, but not vice versa. In what concerns this paper we will consider only the actual world $w_0 = F^{-1}(k)$. Moreover, the set of basic intensional equivalences are designed by users, and we will not verify if they satisfy the modal formula used to define the intensional equivalence: the definition above is of theoretical interest but useful to understand the meaning of the intensional equivalence, and the “omniscient” inference relation $\vdash$, able to deduce all other intensional equivalences from the given basic set.

3 Embedding of P2P database system

P2P systems offer an alternative to traditional client-server systems for some application domains. A P2P system has no centralized schema and no central administration. In P2P systems, every node (peer) of the system acts as both client and server and provides part of the overall information available from an Internet-scale distributed environment. In this paper we consider a formal framework based on the following considerations [21]: what we need here is

- a mechanism that is able, given any two peer databases, to define mappings between them, without resorting to any unifying (global) conceptual structure.
- a completely decentralized network of database peers: all peers serve as entry points for search, offering their relational schema in order to formalize user queries.
- query answering, fundamentally based on interactions which are strictly local and guided by locally defined mappings of a considered peer w.r.t. other peers.
- not limit a-priori the topology of the mapping assertions between peers in the system: we do not impose acyclicity of assertions.

In order to be able to share the knowledge with other peer $P_j$ in the network $\mathcal{N}$, each peer $P_i$ has also an export-interface module $\mathcal{M}_{ij}$ composed by groups of ordered pairs of intensionally-equivalent logical views (conjunctive queries over peer’s ontologies), denoted by $(q_i, q_j)$, or equivalently by $q_i \approx q_j$. Notice that $(q_i, q_j)$ does not mean that $q_i$ logically implicates $q_j$ or vice versa, as in extensional mapping definitions, based on material implication.

**Definition 6.** The P2P network system $\mathcal{N}$ is composed by $2 \leq N$ independent peers, where each peer module $P_i$ is defined as follows: $P_i := (\mathcal{O}_i, \mathcal{M}_i)$, where $\mathcal{M}_i = \langle \mathcal{M}^{i1}, \ldots, \mathcal{M}^{iN} \rangle$ is an interface tuple with $\mathcal{M}^{ij}$, $1 \leq j \leq N$ a (possibly empty) interface to other peer $P_j$ in the network, defined as a group of intensionally equivalent query connections, denoted by $(q_{ij}^{1k}, q_{ij}^{2k})$ where $q_{ij}^{1k}$ is a conjunctive query defined over $\mathcal{O}_i$, while $q_{ij}^{2k}$ is a conjunctive query defined over the ontology $\mathcal{O}_j$ of the connected peer $P_j$:

$$\mathcal{M}^{ij} = \{(q_{ij}^{1k}, q_{ij}^{2k}) \mid 1 \leq k \leq |ij|\},$$

and $|ij|$ is the total number of query connections of the peer $P_i$ toward a peer $P_j$.

Intuitively, when an user defines a conjunctive query over the ontology $\mathcal{O}_i$ of the peer $P_i$, the intensionally equivalent concepts between this peer and other peers will be used in order to obtain the answers from a P2P system.

The formal semantic framework for P2P database systems, presented also in [18] as a
hybrid modal logic, in this paper will be defined as quotiented (by intensional equivalence) intensional FOL. We will consider only the actual world \( w_0 = F^{-1}(k) \), corresponding to the extensionalization function \( k \) of the intensional FOL \( L_\omega \). The actual world for \( L_\omega \) corresponds to the actual extension of peer databases. When an user defines a conjunctive query \( q(x) \) over an ontology \( O_i \) of a peer database \( P_i \), the answer to this query is computed in this actual world \( w_0 \), that is in the actual extension of all peer databases in a P2P network \( N = \{ P_i \mid 1 \leq i \leq N \} \).

**Definition 7.** Let \( N = \{ P_i \mid 1 \leq i \leq N \} \) be a P2P database system. The intensional FOL \( L_\omega \) for a P2P network \( N \) is composed by:

1. The set of basic intensional entities is a disjoint union of entities of peers \( S_I = \bigsqcup_{1 \leq i \leq N} \{ r(y) \mid r(y) \in O_i \} \). The intensional interpretation of the set of all intensional entities define the domains \( D_n, n \geq 1 \);
2. The extensional part of a domain, \( D_{-1} \), corresponds to the disjoint union of domains of peer databases. The intension-in-proposition part, \( D_0 \), is defined as disjoint union of peer’s Herbrand bases;
3. The basic set of the equivalence relation \( =_i \) is defined as follows: for each peer \( P_i \) if \( (q_{ij}^{(1)}, q_{jk}^{(1)}) \in M_{ij} \), then \( q_{ij}^{(1)} =_i q_{jk}^{(1)} \).

The COMPLETE P2P answer to a conjunctive query \( q(x) \) over a peer \( P_i \) is equal to the extension of the quotient-intensional concept \( \langle q(x) \rangle \), whose equivalence class is determined by the deductive omniscient closure of \( \vdash_i \), in the quotient intensional P2P logic \( L_\omega/\approx \).

The quotient intensional FOL \( L_\omega/\approx \) (its algebraic counterpart is a Lindenbaum-Tarski algebra) is fundamental for query-answering in intensional P2P database mapping systems: given a query \( q(x) \) over a peer \( P_i \), the answer to this query is defined as the extension of the denotation of the intensional concept \( \langle q(x) \rangle \), in the intensional P2P logic \( L_\omega/\approx \), obtained by the omniscient inference relation \( \vdash_i \) of this embedding of a P2P database system into the S5 modal intensional FOL \( L_\omega \). While this logical omniscience of the embedding of P2P database systems may be acceptable in the study of theoretically perfect query-agent reasoners, any model or belief with this property will be unacceptable for representing resource and time bounded query-agents.

**Proposition 1** Let \( C(x) \) be a logic formula defined from built-in predicates (ex, \( \leq, \geq \), etc.), then \( \langle A(x) \rangle_\alpha =_i \langle B(x) \rangle_\alpha \) implies \( \langle A(x) \land C(x) \rangle_\alpha =_i \langle B(x) \land C(x) \rangle_\alpha \).

**Proof:** immediately from the fact that a built-in formulae \( C(x) \) has constant extension in any possible world in \( \mathcal{W} \).

4 The non omniscient deduction model for query-agents

In real Web applications we will never have omniscient query-agents that will contemporary have the complete and whole knowledge about all ontologies of all peers. Such supposition would generate the system with a global and centralized knowledge, in contrast with our pragmatic and completely decentralized P2P systems with completely
independent peers, which can change their local ontology in any instance of time without informing any other peer or "global" system about it. Thus, what we will consider for query-agent reasoning system is a weaker form of deduction than ⊢\textsubscript{str} of this ideal omniscient intensional logic inference, more adequate for the limited and local knowledge of query-agents about the peers: what we consider is that a query agent will begin its work for a given user query \( q(x) \) over a peer \( P_i \), and by using only the local knowledge about this peer’s ontology and the set of its local intensional mappings towards other peers, will be able to move to the locally-next peers to obtain answers from them also. This context-sensitive query-answering is analog to the human query answering: interviewer will ask the indicated person and will obtain his known answer, but this person can tell also which other people, he believes, are able to respond to this question also. It will be the task of the interviewer to find other people and to reformulate the question to them. It is, practically impossible to have all people who know something about this question to be in common interaction one with all other to combine the partial knowledge of each of them in order to provide the complete possible answer to such question.

They will be the "bridge" which a query agent can use to rewrite the original user query over a peer \( P_i \) into intensionally-equivalent query over other peer \( P_j \) which has different (and independent) ontology from the peer \( P_i \).

The answers of other peers will be epistemically considered as possible answers because they are based on the belief which has the peer \( P_i \) about the knowledge of a peer \( P_j \): this belief is formally represented by supposition of a peer \( P_i \) that the pair of queries \((q_{ij}^1, q_{ij}^2) \in \mathcal{M}_{ij}\) is intensionally-equivalent.

The context of a peer \( P_i \) represents the whole information contribution of other peers to the local knowledge of this peer, and, consequently, can be used during the query-answering: given any conjunctive query \( \varphi(y) \) over a peer \( P_i \) in a world \( w \), first, we compute the set of certain answers \( k(d(\varphi(x))) \), and after that also the set of intensionally-possible answers obtained from the information contribution from other peers, i.e., from the context \( C(P_i) \) of the considered peer: thus, the answer to a query over a peer is context-dependent. By changing the context of a peer we will obtain different set of possible answers: the syntax of the interface module of a peer is a specification for such context. For each peer there are at least two modalities in order to obtain intensionally-possible answers:

- **Atomic or pure P2P query answering**: in this case each peer can have a number of context, each one for an interface toward some other peer. The query-agent has to try to completely reformulate the original query \( \varphi(y) \) over a peer \( P_i \) for any other peer \( P_j \) if \( \mathcal{M}_{ij} \) is not empty. Then a peer \( P_j \) will be able to respond with its possible answers. In this case we define the context \( C(P_i) = \mathcal{M}_{ij} \).

- **Data integration P2P query answering**: we can consider partial answers from all contextual peers for a given peer \( P_i \), defined in its interface module. The query agent will assemble (join) the partial answers from them in order to obtain possible answers. In this case we define the context as follows:
  \[
  C(P_i) = \{ (q_{i1}(x_1), \{ q_{j1}(x_1) \} \mid (q_{i1}(x_1), q_{j1}(x_1)) \in \mathcal{R}) \mid q_{i1}(x_1) \in \pi_1(\mathcal{R}) \}
  \]
  where \( \mathcal{R} = \bigcup_{1 \leq j \leq N} \mathcal{M}_{ij} \), and \( \pi_1 \) is a first projection.

In the rest of this paper we will consider the atomic case only.
Definition 8. For a given context \( \mathcal{C}(P_i) = \mathcal{M}^{ij} \) we define its Deduction model as follows:

1. It is composed by the logic theory, \( \mathcal{L}_w(P_i) \), obtained by embedding this peer \( P_i \) into Intensional FOL (from Definition 7, without its interface to other peers different from a peer \( P_j \)).

2. For any query \( q(x) \) over a peer \( P_i \), intensionally equivalent to the conjunctive query \( C(x) \land \psi \), where \( \{ v_{i_1}, ..., v_{i_k} \} = \pi_1(\{ (v_{i_1}, v_{j_1}), ..., (v_{i_k}, v_{j_k}) \}) \subseteq \pi_i(\mathcal{M}^{ij}) \) is a subset of views over \( P_i \), and \( C(x) \) is a predicate form over the variables in \( x = (x_1, ..., x_n) \), from built-in predicates only, we introduce in \( \rho \) the following deductive bridge rule:

\[
(\langle \psi \rangle) \equiv \in \langle \psi \rangle, \quad \langle q(x) \rangle = \langle C(x) \land \psi \rangle, \quad \langle q(x) \rangle : P_i
\]

Intuitively, the logic theory \( \mathcal{L}_w(P_i) \subset \mathcal{L}_w \), is the logic of the "pure" peer \( P_i \), with only interface module toward a peer \( P_j \), which for a given query \( q(x) \) over its ontology \( \mathcal{O}_i \), is able to deduce if there is an intensionally equivalent query over its views contained in the interface module \( \mathcal{C}(P_i) = \mathcal{M}^{ij} \): if such deduction exists (in practice for that we can use the perfect query-rewriting algorithms [22]), that is, if \( \mathcal{L}_w(P_i) \vdash \in \langle q(x) \rangle = \langle C(x) \land \psi \rangle, \) then it is possible to use the bridge rule in order to derive, intensionally equivalent to user query, the query \( C(x) \land \psi \) over views of the other peer \( P_j \).

We can introduce the reasoning capabilities of the query-agents, able, for a given user query \( q(x) \) over a peer \( P_i \), to infer other query formulae, intensionally equivalent to \( q(x) \) over the ontologies of other peers in a given P2P database system. We define the 'belief' binary predicate \( B \) which express that agent beliefs: that the first argument (term) of this predicate is intensionally equivalent to the second argument.

Definition 9. In what follows we will define a query-agent with its 'belief' binary predicate \( B \), with the following axioms:

1. For intensionally equivalent queries \( q_i(x) \) and \( q_j(x) \), over peer ontologies of \( P_i \) and \( P_j \), respectively:

\[
\langle q_i(x) \rangle = \in \langle q_j(x) \rangle \Rightarrow \langle B(q_i(x)), q_j(x) \rangle
\]

2. for the symmetric property:

\[
B(<\varphi>, <\psi>) \Rightarrow \langle B(<\psi>, <\varphi>) \rangle
\]

3. for the transitive property:

\[
(\langle B(<\varphi>, <\psi>), B(<\psi>, <\phi>) \rangle) \Rightarrow \langle B(<\varphi>, <\phi>) \rangle
\]

The set of these axioms for a 'belief' predicate, together with the Deductive model of actual query-agent context, will define the weak (non omniscient) deductive inference of this agent, denoted by \( \vdash_\rho \), during the query-answering process.

Let us verify that the 'belief' predicate satisfy the Konolige conditions (K1) and (K2) from the introduction. In fact if \( \mathcal{L}_w(P_i) \vdash_\rho \langle \phi \rangle = \in \langle \psi \rangle \), than from the axiom 1 of Definition 9 and by Modus ponens we obtain that in the actual world \( k(I(\langle B(<\phi>, <\psi>) \rangle) = t \), that is, \( k(\text{pred}(I(\langle B \rangle), d(<\phi>), d(<\psi>)))) = t \) iff \( d(<\phi>), d(<\psi>)) \in k(I(\langle B \rangle)) \).

The deductive closure for 'belief' predicate is maintained only for the formulae expressed over peer ontologies, intensionally equivalent to the given user query, just because we are only interested for the agents for the query-answering in P2P systems. We can also enrich deductive belief capabilities of agents by adding other axioms, as for example:
1. \( B(\langle \phi \rangle \Rightarrow \phi) \), express that the agent believes that what is believed is true.
2. \( B(\langle \phi \rangle) \Rightarrow B(\langle B(\langle \phi \rangle) \rangle) \), express the positive agent’s introspection.

**Example:** Let us consider the cyclic P2P system in a Fig.1, with a two peers: \( P_i \), with the ontology \( O_i \), and the interface \( M^{ui} = \{ (v_{im}, v_{jm}) | v_{im} \approx v_{jm}, \text{and } 1 \leq m \leq k_1 \} \), toward the peer \( P_j \), and the peer \( P_j \), with the ontology \( O_j \), and the interface \( M^{ji} = \{ (w_{jm}, w_{im}) | v_{jm} \approx v_{im}, \text{and } 1 \leq m \leq n_1 \} \), toward the peer \( P_i \). We denote by \( v_{im} \approx v_{jm} \) the intensional equivalence \( \langle v_{im} \rangle \approx_{in} \langle v_{jm} \rangle \). In what follows, the bottom index of a query identifies the peer relative to such a query.

Let \( q_i(x) \) be the original user’s conjunctive query over the ontology \( O_i \) of the peer database \( P_i \), and the query-agent takes its first context \( C(P_i) = M^{ui} = \{ (v_{im}, v_{jm}) | v_{im} \approx v_{jm}, \text{and } 1 \leq m \leq k_1 \} \). Then the agent can apply the axiom 1 in Definition 9, \( \phi \models B(\langle q_i(x) \rangle) \Rightarrow B(\langle q_i(x) \rangle) \), and we have, by Modus ponens,

\[
L_w(P_i) \models \phi \models B(\langle q_i(x) \rangle) \Rightarrow \langle q_i(x) \rangle.
\]

If \( L_w(P_i) \models \phi \models B(\langle q_i(x) \rangle) \Rightarrow \langle q_i(x) \rangle \), then the agent can use the bridge rule

\[
(\langle v_{i1} \rangle \approx_{in} \langle v_{j1} \rangle, \ldots, \langle v_{ik} \rangle \approx_{in} \langle v_{jk} \rangle) \Rightarrow \langle q_i(x) \rangle \Rightarrow \langle q_i(x) \rangle) : P_i
\]

in order to derive, intensionally equivalent to user query, the query \( \psi(v_{j1}, \ldots, v_{jk}) \) equal to conjunctive formula \( C(x) \land v_{j1} \land \ldots v_{jk} \), that is, in the intensional logic language holds the identity \( \text{id}_1 : \langle q_i(x) \rangle \Rightarrow \langle \psi(v_{j1}, \ldots, v_{jk}) \rangle \), or, equivalently, \( \Box(q_i(x) \equiv \psi(v_{j1}, \ldots, v_{jk})) \).

Then the agent can use the bridge rule

\[
\langle v_{i1} \rangle \approx_{in} \langle v_{j1} \rangle, \ldots, \langle v_{ik} \rangle \approx_{in} \langle v_{jk} \rangle, \langle q_i(x) \rangle \Rightarrow \langle \psi(v_{j1}, \ldots, v_{jk}) \rangle)
\]

in order to derive, intensionally equivalent to user query, the query \( \psi(v_{j1}, \ldots, v_{jk}) \) equal to conjunctive formula \( C(x) \land v_{j1} \land \ldots v_{jk} \) over views of the other peer \( P_j \), that is

\[
\langle q_i(x) \rangle \Rightarrow \langle \psi(v_{j1}, \ldots, v_{jk}) \rangle)
\]

**Fig. 1.** Derivation of intensionally equivalent queries

In the next step the query agent, will try to take the next context of the current peer \( P_i \), but there is no other non elaborated context of this peer, so based on the precedent bridge rule, will pass from the peer \( P_i \) to the peer \( P_j \), and will take its context \( C(P_j) = M^{ji} = \{ (w_{jm}, w_{im}) | v_{jm} \approx v_{im}, \text{and } 1 \leq m \leq n_1 \} \).

We assume that, when the agent changes the context, all intensional equivalences where, at least one argument is a query formula contained in its belief set \( B \), are preserved together with its belief set in \( B \).
If $\mathcal{L}_\omega(P_1) \vdash _\rho <q_j(x)>, \text{ for some query formula } q_j(x) \text{ over the ontological type of peer } P_j$, then

(b) $\mathcal{L}_\omega(P_1) \vdash _\rho <q_j(x)> =_{in} <\Psi(v_{j1}, \ldots, v_{jk})>$

(From the intensional identity we have that $\Box[q_j(x)] \equiv <\Psi(v_{j1}, \ldots, v_{jk})>$, and from S5 modal intensional FOL we deduce $\mathcal{L}_\omega(P_1) \vdash _\rho q_j(x) \equiv <\Psi(v_{j1}, \ldots, v_{jk})>$ (by modal rule of necessitation), so that $\mathcal{L}_\omega(P_1) \vdash _\rho \diamond q_j(x) \equiv <\Psi(v_{j1}, \ldots, v_{jk})>$, i.e., $\mathcal{L}_\omega(P_1) \vdash _\rho <q_j(x)> =_{in} <\Psi(v_{j1}, \ldots, v_{jk})>$, and by the symmetry and transitivity property of the equivalence relation, from (a) and (b) we obtain $\mathcal{L}_\omega(P_1) \vdash _\rho <q_i(x)> =_{in} <q_j(x)>$, and by the axiom 1 in Definition 9).

(2) $\mathcal{L}_\omega(P_1) \vdash _\rho B[q_i(x)>$, $\Leftrightarrow q_j(x)>$, and by the axiom 2 in Definition 9

(3) $\mathcal{L}_\omega(P_1) \vdash _\rho B[q_j(x)>$, $\Leftrightarrow q_i(x)>$, and from (3) and (2) and the axiom 3 in Definition 9

(4) $\mathcal{L}_\omega(P_1) \vdash _\rho B[q_j(x)>$, $\Leftrightarrow q_i(x)>$.

This query-agent reasoning corresponds to the top-horizontal arrow in Fig.1. Analog process can be described for the bottom-horizontal arrow in Fig.1.

At the end of deduction the set of intensionally equivalent queries over peers in a P2P database system is equal to the first (or second) projection of the agent’s belief set $B$. This set of queries, in this example $\pi_1(B) =\{<q_i(x)> \Leftrightarrow q_j(x)>\}$, is the subset of the equivalent class $C$ for the given user query, which in the intensional FOL $\mathcal{L}_\omega/\omega$ is represented by the quotient intensional entity $Q(x)$, whose extension (from Definition 5) in the actual world $W_0$ is defined by $\mathcal{F}(w_0)(d(<Q(x)>)) = \{t \in \mathcal{D} \mid A_1(t) \land A_i(x) \in C\} = \bigcup_{1 \leq i \leq m} \mathcal{F}(w_0)(d(<A_i(x)>))$, that is, the union of answers for query-agent’s derived queries is a subset of the extension of this quotient-intensional entity $Q(x)$.

\[ \square \]

Remark: This query-answering process is valid also for the union of conjunctive queries: in fact, given two intensional equivalences between conjunctive queries, $<q_{i1}> =_{in} <q_{i2}>$ and $<q_{j1}> =_{in} <q_{j2}>$, holds that $<q_{i1} \land q_{j2}> =_{in} <q_{i2} \land q_{j1}>$; from the fact that S5 modal intensional FOL is a normal modal logic where holds that $\Diamond(A \lor B) \equiv \Diamond A \lor \Diamond B$.

Theorem 1: The deductive inference of a query-agent, $\vdash _\rho$, is a sound weakening of the omniscient deductive inference $\vdash _m$ in the S5 modal intensional FOL, for the P2P database system $\mathcal{L}_\omega$ with the peer databases which does not contain the integrity constraints in the form of negative clauses $\neg A_1 \lor \ldots \lor \neg A_m$, $m \geq 2$.

Proof. By structural induction on the number of conjuncts in the expression: it is enough to prove for expressions composed by two conjuncts. Let us define $\text{lub}(\phi(x)) = \bigvee_{\psi \in \text{R}} \mathcal{F}(w)(d(<\phi(x)>)), so that we have $<\phi(x)> =_{in} <\phi_1(x)> \text { iff } \Diamond \phi(x) \equiv \Diamond \phi_1(x)$ if $\text{lub}(\phi(x)) = \text{lub}(\phi_1(x))$.

Let $b_1, b_2$ be any two (virtual) predicates over a peer $P_1$, $q_{i1}(x, y)$ and $q_{i2}(y, z)$ respectively, and $c_1, c_2$ (equal to $q_{j1}(x, y)$ and $q_{j2}(y, z)$ respectively) any two (virtual) predicates over a peer $P_j$, such that $<b_1> =_{in} <c_i>, \ i = 1, 2$. We have to prove that $\text{lub}(\phi(x, z)) = \text{lub}(\psi(x, z))$, where $\phi(x, z) \equiv (q_{i1}(x, y) \land q_{i2}(y, z))$ and $\psi(x, z) \equiv (q_{j1}(x, y) \land q_{j2}(y, z))$. 
From the facts that \( \text{lub}(q_{11}(x, y)) = \text{lub}(q_{11}(x, y)) \) and \( \text{lub}(q_{22}(y, z)) = \text{lub}(q_{22}(y, z)) \), we define the set

\[ S_L = \{(a, c) | \exists b.((a, b) \in \text{lub}(q_{11}(x, y)) \land (b, c) \in \text{lub}(q_{22}(y, z)))\} \]

\[ = \{(a, c) | \exists b.((a, b) \in \text{lub}(q_{11}(x, y)) \land (b, c) \in \text{lub}(q_{22}(y, z)))\} \]

Let us prove that \( \text{lub}(\varphi(x, z)) = \bigcup_{w \in W} \{((a, c) | \exists b.((a, b) \in \mathcal{F}(w)(d(<q_{11}(x, y)>)) \land (b, c) \in \mathcal{F}(w)(d(<q_{22}(y, z)>)))\} \) is equal to \( S_L \). First, from \( \mathcal{F}(w)(d(<q_{11}(x, y)>)) \subseteq \text{lub}(q_{11}(x, y)), k = 1, 2 \) holds that \( \text{lub}(\varphi(x, z)) \subseteq S_L \).

Let us prove, that also \( \text{lub}(\varphi(x, z)) \supseteq S_L \), i.e. that for any \((a, b) \in S_L\) also \((a, b) \in \text{lub}(\varphi(x, z))\). Let us suppose that there is one \((a, c)\) such that \((a, c) \in S_L\) but \((a, c) \notin \text{lub}(\varphi(x, z))\), i.e., that for all possible worlds for this P2P system, \( w \in W \), holds that \( \pi_2(\mathcal{F}(w)(d(<q_{11}(a, y)>))) \cap \pi_1(\mathcal{F}(w)(d(<q_{22}(y, c)>))) = \{\} \) (is empty), where \( \pi_1, \pi_2 \) are the first and the second projections. That is, the following logic formula must hold

\[ \neg q_1(a, y) \lor \neg q_2(y, c) \lor \neg(y = y') \].

But such constraint (negative clause) cannot exist in this class of peers, thus the supposition is false, and we conclude that \( S_L = \text{lub}(\varphi(x, z)) \).

By the same way we obtain that \( S_L = \text{lub}(\psi(x, z)) \), thus \( <\varphi(x, z)> =_m <\psi(x, z)> \), and the bridge rules of the query-agents

\[
\begin{align*}
\langle v_{i_1} \rangle =_m \langle v_{j_1} \rangle, \ldots, \langle v_{i_k} \rangle =_m \langle v_{j_k} \rangle, \quad \langle q(x) \rangle = \langle C(x) \land v_{i_1} \land \ldots \land v_{i_k} \rangle : P_1 \\
\langle q(x) \rangle = \langle C(x) \land v_{j_1} \land \ldots \land v_{j_k} \rangle : P_2
\end{align*}
\]

are valid deductions also for the omniscient S5 modal intensional FOL \( L_\omega \) of a P2P system: thus \( \vdash_m \) is a sound weakening of the omniscient deductive inference \( \vdash_m \).

(Easy to verify: from \( \langle v_{i_1} \rangle =_m \langle v_{j_1} \rangle, \ldots, \langle v_{i_k} \rangle =_m \langle v_{j_k} \rangle \) we conclude that

\( \langle v_{i_1} \land \ldots \land v_{i_k} \rangle =_m \langle v_{j_1} \land \ldots \land v_{j_k} \rangle \), and from Proposition 1 we obtain that \( \langle C(x) \land v_{i_1} \land \ldots \land v_{i_k} \rangle \); from the fact that, \( \langle q(x) \rangle = \langle C(x) \land v_{i_1} \land \ldots \land v_{i_k} \rangle \) implies \( \langle q(x) \rangle =_m \langle C(x) \land v_{j_1} \land \ldots \land v_{j_k} \rangle \), and the transitivity of \( =_m \) we deduce that \( \langle q(x) \rangle =_m \langle C(x) \land v_{j_1} \land \ldots \land v_{j_k} \rangle \).)

5 Conclusion

The problem of defining the semantics for intensional mappings between ontologies of peer-databases can be specified in a S5 modal intensional FOL language: such modal logic has omniscient deductive inference which is not adequate for real-life query-agents in Web based P2P systems, because in such systems there is no any global schema accessible instantly by query-agents, but only local intensional mappings between peers. Moreover, we supposed that a single query-agent is resource-bounded believer, which can not contain contemporary the knowledge of all (possibly thousands) database peers, so that can not use the omniscient deductive inference of the logic theory of the whole P2P system. We defined the non omniscient deductive model for query-agents, based on the contextualization of the P2P systems w.r.t. peer’s interface modules, with the bridge rules for such contextual reasoning. We model query-agents, which are initiated by the user query over the ontology of an initial peer database, with a ‘belief’ predicate for intensionally equivalent queries over peers in a P2P system, deductively inferred by agent w.r.t. the given user query over some peer database, during the query-answering process when agents move from one to other context by
using local peer’s P2P mappings. This belief of a query-agent reflects the epistemic uncertainty of mappings between peers: each peer is a completely free to change its local knowledge and ontology without any communication to other peers, so that the intensional mappings, initially defined by software developers, can not be more fully adequate in the future; alternatively, in the case of the dynamic, algorithmically derived intensional ontology mappings between two peers, they will be only approximative, so, the only certain answers to user query we obtain are from the directly interrogated peer database, while the answers from other peers can be considered as possible answers. Thus, w.r.t. the classic logic the query-answering of this system is only sound but not complete. The sound and complete query-rewriting answering based on this intensional non standard logic is presented in [23] instead.

References

